A Study of Elementary Excitations of Liquid Helium-4 Using Macro-orbital Microscopic Theory

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Abstract

Energy of elementary excitations $[E(Q)]$ and the anomalous nature of small Q phonons in $He-II$ are studied by using our macro-orbital microscopic theory of a system of interacting bosons (*cond-mat/0606571*). It is observed that: (i) the experimental $E(Q) = E(Q)_{exp}$ of $He-II$ not only agrees with our theoretical relation, $E(Q) = \hbar^2 Q^2/4mS(Q)$, but also supports an important conclusion of Price that $S(0)$ should have zero value for quantum fluids, and (ii) Feynman's energy of excitations $E(Q)_{Fyn} = \hbar^2 Q^2/2mS(Q)$ equals $\approx 2E(Q)_{exp}$ even at low Q. Three problems with the Feynman's inference that $E(Q)_{Fyn}$ has good agreement with $E(Q)_{Exp}$ at low Q are identified. It is argued that the theory can also be used to understand similar spectrum of the BEC state of a dilute gas reported by O'Dell *et. al.*

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1. Introduction

Elementary excitations of *Liquid* ^4He, -a typical *system of interacting bosons* (SIB), have been a subject of extensive theoretical and experimental investigation which have been elegantly reviewed in [1]. While experimental studies, performed under different physical conditions, are reported in [2-9], development of their microscopic understanding started with Feynman [10] who obtained

$$E(Q) = E(Q)_{Fyn} = \frac{\hbar^2 Q^2}{2mS(Q)} \tag{1}$$

where $E(Q)$ and $S(Q)$, respectively, represent the energy of excitations and structure factor of the system at wave vector Q. Guided by the observation, that $E(Q)_{Fyn}$ has close agreement with experimental $E(Q)[= E(Q)_{exp}]$ of $He-II$ at low Q and equals $\approx 2E(Q)_{exp}$ at high Q, Feynman and Cohen [11] used the phenomenon of back flow to obtain a relation of better agreement with $E(Q)_{exp}$ but the difference at higher Q could not be reduced to desired level. Consequently, several researchers [12-14] used different mathematical tools and computational techniques to find better agreement between theory and experiments.

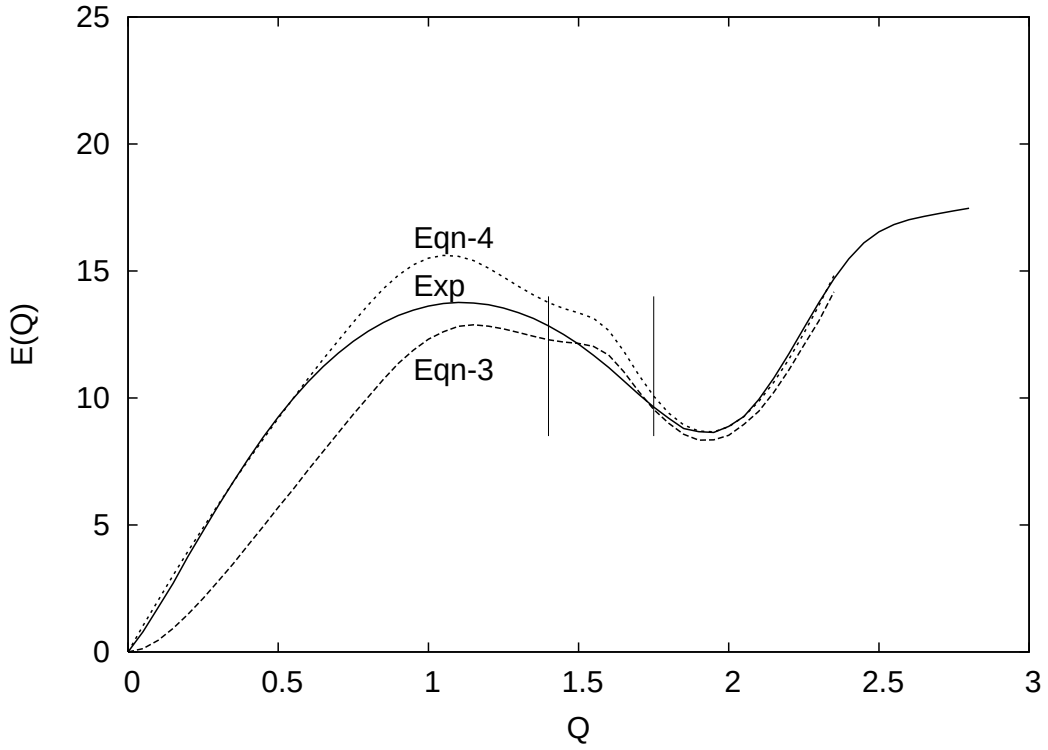

Figure 1. $E(Q)$ (in ^{o}K) vs Q (in Å^{-1}) curves for $He-II$. While 'Exp' identifies $E(Q)_{exp}$, 'Eqn-3' and 'Eqn-4' represent $E(Q)_{mo}$ and $E(Q)^*_{mo}$ obtained by using Eqns. 3 and 4, respectively. Both $E(Q)_{mo}$ at Q values between two small vertical lines seem to have deviations from their normal trend.

Using macro-orbital representation of a particle in a many body system [15], we developed the long awaited microscopic theory of a SIB [16] that explains the properties of liquid ^4He with unparalleled simplicity, clarity and accuracy. Clubbing Feynman's approach of concluding Eqn. 1 with macro-orbitals, we obtained [16]

$$E(Q) = E(Q)_{mo} = \frac{\hbar^2 Q^2}{4mS(Q)} \tag{2}$$

This paper compares Eqn. 2 with $E(Q)_{exp}$ of $He-II$ and identifies important aspects of an effective $S(Q)$ to be used in this equation. In addition, it also analyzes the basic factors responsible for the experimentally observed anomalous nature of small Q phonons of $He-II$ by using another important result of our theory (*cf.*, Eqn. 5, below).

2. Elementary Excitations of He-II

In order to understand the spectrum of elementary excitations of $He-II$ in terms of Eqn. 2, we use $S(Q)_{exp}$ and $E(Q)_{exp}$ compiled by Donnelly and Barenghi [17]. Appendix

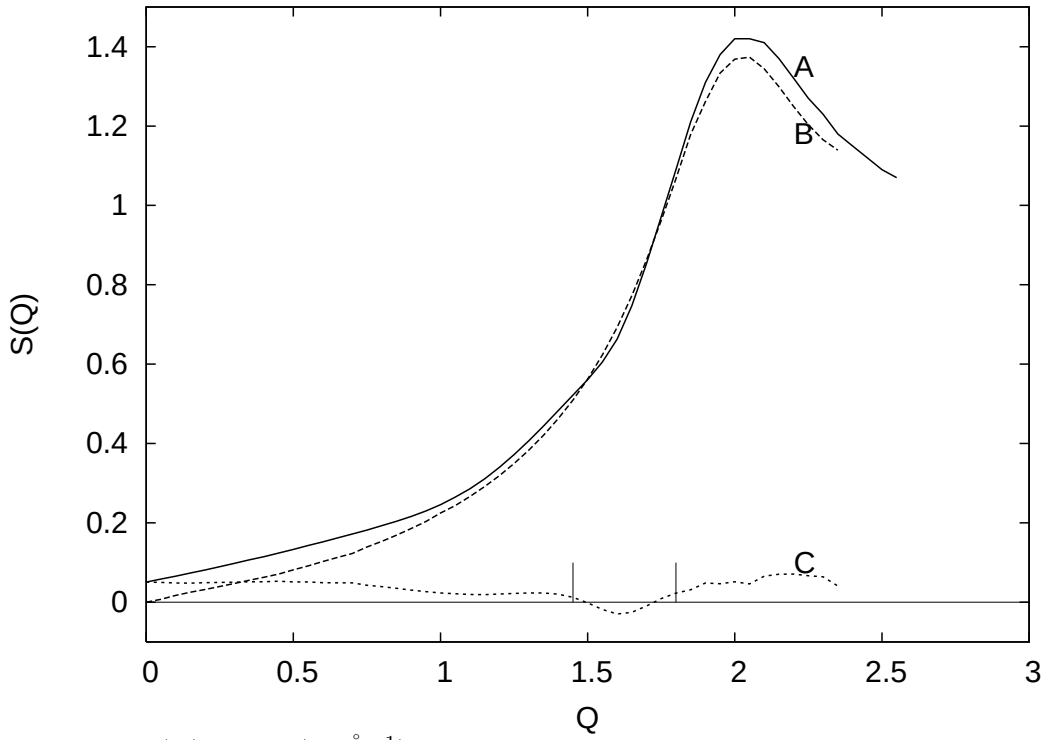

Figure 2. $S(Q)$ vs Q (in Å^{-1}) curve for liquid helium-4. While Curve-A depicts $S(Q)_{exp}$ (Appendix I, column 2), Curve-B represents $S(Q)'$ (Appendix I, column 3) which, when used in Eqn. 2, reproduce $E(Q)_{exp}$ (Appendix I, column 5) exactly and Curve-C shows $\Delta S(Q) = S(Q)_{exp} - S(Q)'$ (Appendix I, column 4)).

I reproduces required $S(Q)_{exp}$ and $E(Q)_{exp}$ data for ready reference (*cf.* columns 2 and 5, respectively). Using $S(Q) = S(Q)_{exp}$ in Eqn. 2, we obtain

$$E(Q)_{mo} = \frac{\hbar^2 Q^2}{4mS(Q)_{exp}} \tag{3}$$

which are tabulated in Column 6 of Appendix I and plotted as curve 'Eqn-3' in Figure 1 where $E(Q)_{exp}$ is also plotted as curve 'Exp' for comparison. We find that $E(Q)_{mo}$ does not have (i) desired level of agreement with $E(Q)_{exp}$ [the $\Delta E(Q) = E(Q)_{exp} - E(Q)_{mo}$ is as large as $\approx E(Q)_{mo}$ at $Q \approx 0.35\text{Å}^{-1}$ (*cf.* Appendix I) and the ratio, $R = E(Q)_{exp}/E(Q)_{mo}$ tends to have infinitely large value when $Q \to 0$], (ii) agreement in the basic nature of the Q dependence of $E(Q)_{exp}$ ($\propto Q$) and $E(Q)_{mo}$ ($\propto Q^2$) and (iii) consistency with experimentally observed *non-zero value* ($\approx 238.21 m/sec$) of the group velocity (V_g) and phase velocity (V_p) of low Q (0 to $\approx 0.5\text{Å}^{-1}$) phonons in $He-II$. The following analysis concludes that the absence of '(i)' to '(iii)' arises due to non-zero value [0.051] of $S(0)$ adopted in [17].

It is evident that $E(Q)_{exp}$ at low Q is basically a linear function of Q which, however, fits with Eqn. 2 only if: (a) $S(0)$ is exactly zero, and (b) the major Q dependent term

in the series expansion of $S(Q)$ at low Q is $\propto Q$. While the $S(Q)_{exp}$ data for $He-II$ (*cf.* column 2, Appendix I and Figure 2) seem to satisfy '(b)' to a good approximation, but $S(0) = 0.051$ contradicts '(a)'. In case $S(0) \neq 0$, Eqn. 2 (under the limit $Q \to 0$) renders $E(Q)_{mo} \propto Q^2$ concluding $V_g = V_p = 0$ at $Q = 0$ which are in total disagreement with experiments. Evidently, we need $S(0) = 0$ in place of $S(0) = 0.051$ adopted in [17]. In this context we *underline* an important theoretical inference of Price [18] which states that $S(0)$ should be zero for a quantum fluid at *zero* temperature (T). While the importance and validity of this inference have been emphasized recently by Lamacraft [19], our theoretical inference that structural configuration of a quantum fluid at $T \leq T_\lambda$ does not differ from that at $T = 0$ [16] implies that $S(0)$ should be *zero* at all T below $\lambda-$point and this is corroborated by the experimental fact that $E(Q)$ of $He-II$ hardly depends on T. Evidently, correcting $S(Q)_{exp}$ taken from [17], for $S(0)(= 0.051)$, we shift the zero level of $S(Q)$ to $S(Q) = 0.051$ and define $S(Q)^* = S(Q)_{exp} - S(0)$ as the effective $S(Q)$ that should enter in Eqn. 2 to find

$$E(Q)^*_{mo} = \frac{\hbar^2 Q^2}{4mS(Q)^*} = \frac{\hbar^2 Q^2}{4m[S(Q)_{exp} - S(0)]} \tag{4}$$

as a more appropriate relation to determine $E(Q)$ of a SIB. It is obvious that $S(Q)^*$ retains the Q dependence of $S(Q)_{exp}$ including its linearity in Q at low Q. We plot $E(Q)^*_{mo}$ in Figure 1 (*cf.* curve 'Eqn-4') for its comparison with $E(Q)_{exp}$. We note that the maximum difference in $E(Q)^*_{mo}$ and $E(Q)_{exp}$, found to be ≈ 1.8 at 1.1Å^{-1}, is about 13% of $E(Q)_{exp} \approx 13.8$ which compares with the fractional deviation of the recommended values of $E(Q)_{exp}$ and $S(Q)_{exp}$ [17] from their adopted database since, as reported in [17], these deviations are as large as 3% for $E(Q)_{exp}$ and 10% for $S(Q)_{exp}$. To this effect we also determined $S(Q)' = \hbar^2 Q^2/4mE(Q)_{exp}$ which when used in Eqn. 2, obviously reproduce $E(Q)_{Exp}$. We tabulate $S(Q)'$ in Column 3 of Appendix I with its deviation from $S(Q)_{exp}$ [*i.e.*, $\Delta S(Q) = S(Q)_{exp} - S(Q)'$] in Column 4 and plot $S(Q)_{exp}$, $S(Q)'$ and $\Delta S(Q)$ in Figure 2 as Curves A, B and C, respectively. We note that: (i) $S(Q)_{exp}$ (Curve A) and $S(Q)'$ (Curve B) match closely with each other except for some differences at low Q which can be attributed to $S(0) = 0.051$ adopted with $S(Q)_{exp}$ [17] and (ii) the $\Delta S(Q)$ values (Curve C) are positive for most Q points (except for points in the small range, $Q = 1.5$ to 1.7Å^{-1}, where $\Delta S(Q)$ are negative) with average $\Delta\bar{S}(Q) \approx 0.033$ which justifies our use of $S(Q) = 0.051$ as the zero line of $S(Q)^*$. A careful examination of the recommended $S(Q)_{exp}$ data [17] also reveals that the portion of their curve-A (Fig 2) (from $Q = 0$ to $Q \approx 1.9\text{Å}^{-1}$) has three linear segments: (a) from $Q = 0$ to $Q \approx 0.9\text{Å}^{-1}$, (b) from $Q \approx 0.9\text{Å}^{-1}$ to $Q \approx 1.5\text{Å}^{-1}$, and (c) from $Q \approx 1.5\text{Å}^{-1}$ to $Q \approx 1.9\text{Å}^{-1}$ joined smoothly with each other; this can be seen more clearly by plotting $\partial_Q S(Q)_{exp}$ which we have not shown here. This, naturally, implies that $S(Q)_{exp}$ data [17] have some systematic errors. In this context it may be noted that : (i) $E(Q)_{mo}$ (curve 'Eqn-3', Fig. 1) and $E(Q)^*_{mo}$ (curve 'Eqn-4') have deviations from their normal trend only in the region from $Q = 1.45\text{Å}^{-1}$ to $Q = 1.75\text{Å}^{-1}$ (identified by two vertical lines in Figs 1 and 2) where most $\Delta S(Q)$ are negative, and (ii) the $S(Q)_{exp}$ data obtained from x-ray diffraction differ significantly from those obtained from neutron diffraction [17]. The $\Delta S(Q)$ data [falling between -0.0293

and 0.0733 (Appendix I) around $\Delta\bar{S(Q)} \approx 0.033$] also indicate that possible errors of $S(Q)_{exp}$ data [17] could be as large as $\Delta\bar{S(Q)}$. Evidently, within the limits of such errors of $S(Q)_{exp}$ used here, the differences in our theoretical results (Curve 'Eqn-4', Fig 1) and experimental results (Curve 'Exp', Fig 1) are small and this establishes that Eqn. 4 (or Eqn. 2 if the available $S(Q)$ data have $S(0) = 0$ level) are accurate enough to provide $E(Q)$ of a SIB.

In the light of what has been inferred from the preceding discussion, it is evident that $E(Q)_{Fyn}[= 2E(Q)_{mo}]$ equals $\approx 2E(Q)_{exp}$ values not only at high Q but at all Q if we enter $S(Q)^*$ for $S(Q)$ in Eqn. 1. However, if we use $S(Q) = S(Q)_{exp}$ as compiled in [17] where $S(0) \neq 0$, $E(Q)_{Fyn}$ shows three problems: (i) inconsistency with non-zero values of V_g and V_p at $Q = 0$ as concluded above for $E(Q)_{mo}$ (Eqn.3), (ii) disagreement with $E(Q)_{exp}$ even at low Q, and (iii) difference in the Q dependence of $E(Q)_{exp}$ and $E(Q)_{Fyn}$ at low Q. To this effect we determine $E(Q)_{Fyn}$ by using $S(Q)_{exp}$ for $S(Q)$ in Eqn. 1 and compare it with $E(Q)_{exp}$ at few points of low Q values in Table 1.

Table 1 : Comparison of $E(Q)_{Fyn}$ and $E(Q)_{exp}$

Q	$E(Q)_{Fyn}$	$E(Q)_{exp}$	$R = \frac{E(Q)_{exp}}{E(Q)_{Fyn}}$
0.05	0.258	0.804	$\approx$ 3:1
0.10	0.918	1.747	$\approx$ 2:1
0.15	1.852	2.757	$\approx$ 3:2
0.20	2.794	3.772	$\approx$ 4:3
0.25	4.222	4.772	$\approx$ 6:5
0.30	5.564	5.749	$\approx$ 1:1
0.35	6.938	6.694	$\approx$ 1:1
0.40	8.430	7.598	$\approx$ 1:1
0.50	11.380	9249	$\approx$ 5:6

Notably the finding, that the difference in the values of $E(Q)_{Fyn}$ and $E(Q)_{exp}$ at these Q is not significantly large, has been the basis of the inference that $E(Q)_{Fyn}$ agrees with $E(Q)_{exp}$ at low Q. However, a careful analysis of their relative magnitude ($R = E(Q)_{exp}/E(Q)_{Fyn}$) reveals that R at $Q = 0.05\text{Å}^{-1}$ is as high as *three* and it is expected to be still higher for $Q < 0.05$ because, to a good approximation at low Q, $E(Q)_{exp} \propto Q$ and $E(Q)_{Fyn} \propto Q^2$ which render R to go as 1/Q with $Q \to 0$ and hence implies that R assumes infinitely high value at $Q = 0$; in this context it may be noted that $E(Q)_{Fyn}$ (Eqn. 1) goes as Q^2 at low Q when $S(0) \neq 0$. In summary $E(Q)_{Fyn}$ either equals $\approx 2E(Q)_{exp}$ at all Q if $S(Q)_{exp}$ data conform to $S(0) = 0$ or its apparent agreement with $E(Q)_{exp}$ at low Q as observed by Feynman is plagued with above stated three problems.

3. Anomalous Nature of Phonon Velocities at Low Q

The experimentally observed V_g and V_p of phonons in $He-II$ are found to be increasing functions of Q as shown, respectively, by Curves E1 and E2 in Figure 3; in this context

it may be mentioned that experimental V_g are taken from [20], while $V_p = E(Q)/Q$ are obtained from experimental $E(Q)$ taken from [17]. It is evident that this nature of V_g and V_p, limited to small Q, is opposite to the normal trend of V_g and V_p of phonons in crystals. Various studies of this anomalous character of phonons in $He-II$ has been reviewed by Sridhar [21]. In this section we examine how this could be understood in the framework of our microscopic theory [16]. However, to this effect we do not use our $E(Q)^*_{mo}$ data which do not have the required level of accuracy due to different possible errors in $S(Q)_{exp}$ used in Eqn. 4. In stead we use another result of our theory which concludes that the excitations in $He-II$ type SIB for a large range of $Q < Q_{rot}$ (the roton wave vector) can be identified as the phonons (longitudinal acoustic modes) of a chain of identical atoms (a kind of 1-D crystal) presumed to interact through nearest neighbor interaction. In exact agreement with experiments, one expects only a single phonon branch of longitudinal modes because the liquid ^4He is an *isotropic* system where atoms experience no *shear force* to sustain transverse modes. In addition as argued in [16], phonons in this chain can have anomalous nature at small Q if its parameters (*viz.* nearest neighbor interaction constant C and nearest neighbor distance d) in its phonon excitation state differ from those in its ground state and this difference increases with increasing Q. This can happen the way moments of inertia (*i.e.*, the structural parameters of a molecule) in rotationally excited state differ from those in the ground state and the difference increases with increasing energy of rotational excitation. Evidently, the phonon dispersion $\omega(Q) = E(Q)/\hbar$ of the assumed chain of ^4He atoms in $He-II$ can be obtained from

$$\omega(Q) = \sqrt{\frac{4C(Q)}{m}} |\sin\frac{Qd(Q)}{2}| \tag{5}$$

where $C \equiv C(Q)$ and $d \equiv d(Q)$ are assumed to be smoothly varying functions of Q. It may be argued that $C(Q)$ should increase and $d(Q)$ should decrease with Q. We note that the particles, having increased energy under a phonon excitation, have smaller quantum size leading them to come closer to each other which, obviously, means that $d(Q) \leq d(0)$ and $C(Q) \geq C(0)$ and they should, respectively, be the decreasing and increasing functions of Q. However, since the explicit Q dependence of $C(Q)$ and $d(Q)$ is not known we assume

$$C(Q) = C(0) + C' \sin\frac{\pi Q}{2Q_{max}} = 3.0554 + 2.5517 \sin\frac{\pi Q}{2Q_{max}} \tag{6}$$

and

$$d(Q) = d(0) - d' \sin\frac{\pi Q}{2Q_{max}} = 3.5787 - 0.7484 \sin\frac{\pi Q}{2Q_{max}} \tag{7}$$

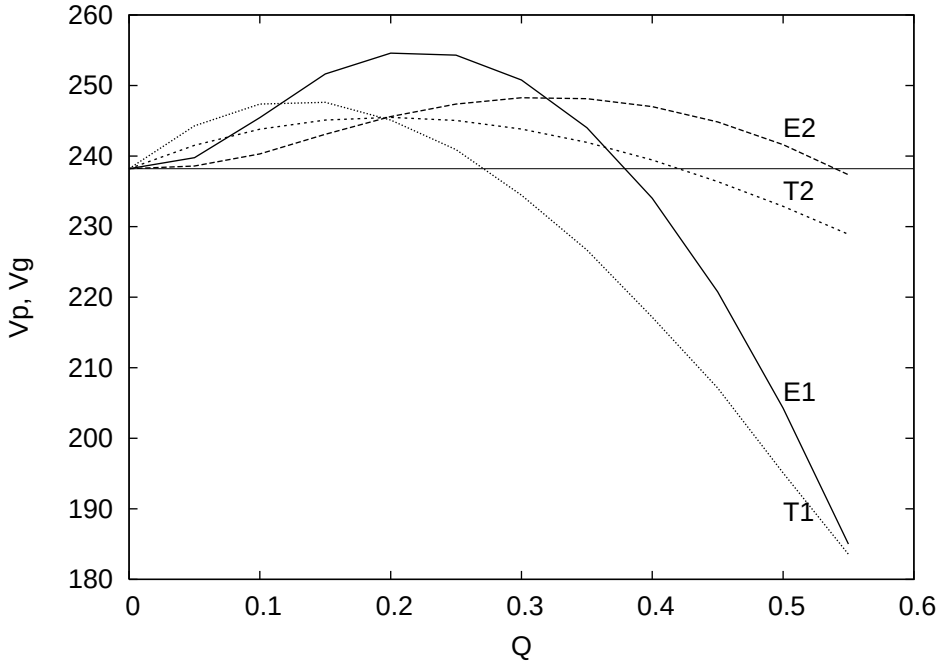

Figure 3. Anomalous variation of V_g and V_p (in m/sec) of phonons with Q (in Å^{-1}) for $He-II$. While curves T1 and E1, respectively, represent theoretical and experimental V_g, curves T2 and E2 are those of V_p.

Table 2 : Theoretical and experimental V_g and V_p in m/sec^*; correspondence with Figure 3 can be set by using T$\equiv$ *theoretical*, E$\equiv$ *experimental*, 1$\equiv V_g$, and 2$\equiv V_p$.

Q	$V_g(th)$	$V_g(exp)$	$V_p(th)$	$V_p(exp)$
0.00	238.21	238.21	238.21	238.21
0.05	244.29	239.78	241.52	238.59
0.10	247.37	245.44	243.80	240.31
0.15	247.62	251.64	245.10	243.08
0.20	245.11	254.59	245.49	245.60
0.25	240.91	254.31	245.04	247.38
0.30	234.46	250.78	243.83	248.26
0.35	226.69	244.02	241.94	248.14
0.40	217.16	234.02	239.44	247.01
0.45	207.13	220.76	236.39	244.84
0.50	195.09	204.27	232.87	241.62
0.55	183.56	185.00	228.92	237.36

The relations (Eqns. 6 and 7) are so assumed to ensure smooth variation of $C(Q)$ and $d(Q)$ from $Q = 0$ to $Q > Q_{max}$ (maxon wave vector) with no discontinuity even in their first Q derivative at Q_{max} beyond which they become Q independent with $d(Q \geq Q_{max}) = d(Q_{max})$ and $C(Q \geq Q_{max}) = C(Q_{max})$. For $He-II$, we fix $C(0)$, C', $d(0)$ and

d' empirically by using certain experimental data. For example $d(0) = 3.5787\mathring{A}$ is fixed by using $He - II$ density $= 0.1450gm/cc$ [22], while fixation of $C(0) = 3.0554dyne/cm$ uses $Q = 0$ phonon velocity $= 238.21\ m/sec$. Similarly, the smallest $d[= d(0) - d']$ is equated to the hard core size (σ) of particles since two particles can not have $d < \sigma$. We obtained $\sigma = 2.8293\mathring{A}$ for ^4He atoms by assuming that phonon energy for the chain having $d = \sigma$ should have its maximum at $Q = Q_{max}$ which equals 1.11 $\mathring{A}^{-1}$ for $He - II$ and this means that $\sigma = \pi/1.11\mathring{A}^{-1}$ and $d' = 0.7484\mathring{A}$. Likewise we use $E(Q_{max}) = 13.92K$ [8] and $d(Q_{max}) = \sigma$ to fix highest $C(Q)[= C(0) + C'] = 5.6071dyne/cm$ and $C' = 3.5517dyne/cm$. This concludes that Eqns. 6 and 7 can help in determining $C(Q)$ and $d(Q)$ to obtain $E(Q)$ at any Q ranging from $Q = 0$ to $Q = Q_{max}$. To obtain $E(Q > Q_{max})$, it is assumed that the chain parameters for $Q \geq Q_{max}$ do not differ from $C(Q_{max}) = 5.6071$ and $d = \sigma$. Using Eqns. 6 and 7 in Eqn. 5, we calculated $E(Q)$, V_g and V_p. While $E(Q)$ so obtained is tabulated as $E(Q)_{eq5}$ in column 8 of Appendix I, V_g and V_p are tabulated in Table 2 (*cf.*, columns 2 and 4, respectively). We plot our theoretical V_g and V_p in Figure 3 (*cf.* Curves T1 and T2, respectively) along with their experimental values (*cf.* Curves E1 and E2, respectively). It is important to note that our theoretical V_g and V_p exhibit anomalous nature of phonons as shown by experiments and their values match closely with experimental values. While a maximum quantitative difference of $\approx 5\%$ for V_g and $\approx 3\%$ for V_p is not very significant but a qualitative difference in the nature of their variation near $Q \approx 0$ indicates that the present choice of the Q dependence of $C(Q)$ and $d(Q)$ (Eqns. 6 and 7) need to be replaced by a better choice. We hope to find the desired choice in our future course of studies.

We plot our theoretical $E(Q)_{eq5}$ (*cf.* column 8 of Appendix I), obtained from Eqn. 5, in Figure 4 (Curve A) along with a plot of $E(Q)_{exp}$ (Curve B). The fact that these two $E(Q)$ match very closely for all Q ranging from $Q = 0$ to Q close to Q_{rot} implies that $E(Q)_{exp}$ of $He-II$ over this range can, really, be accounted for by the phonons of a chain of identical atoms clubbed with what we may call as 'final state effect'. Interestingly, recasting Eqn. 4 as

$$E(Q)_{mo} = \frac{\hbar^2 Q^2}{2m^*} \tag{8}$$

one may identify that the excitations of a SIB like liquid ^4He are nothing but the motions of a single particle whose mass changes with Q as $m^* \approx 2m[S(Q)_{exp} - S(Q = 0)]$ which can be known as effective mass. We note that m^* for $He - II$ changes from $m^* = 0$ at $Q = 0$ to $m^* \approx m$ for $Q >> Q_p$ (the lowest Q of plateau modes) through its maximum value of $\approx 2.74m$ for $Q \approx Q_{rot}$ which falls close to the Q of the peak in $S(Q)$. However, in the light an excellent agreement between $E(Q)_{eq5}$ and $E(Q)_{exp}$ (Figure 4), it will be more appropriate to state that excitations in liquid ^4He are : (i) phonon like for $Q \leq 2\pi/d$, (ii) phonon dressed single particle motions for $2\pi/d \leq Q \leq Q_p$, and (iii) an almost free single particle motion for $Q > Q_p$; note that $Q = 2\pi/d$ and $Q = Q_p$ should not be considered as sharply dividing Q points of the three ranges. It may be argued that this shift in the nature of the excitations arises from the facts that: (i) particles in liquid ^4He

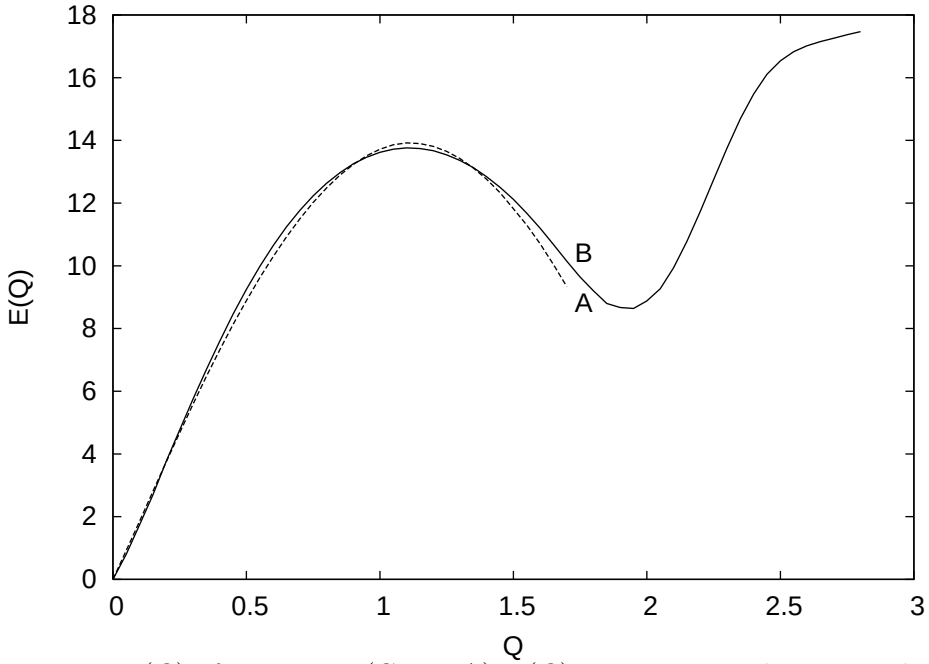

Figure 4. $E(Q)$ of $He-II$: (Curve-A) $E(Q)_{eq5}$ represents the energy dispersion of phonons in a chain of identical atoms whose parameters change with Q and (Curve-B) $E(Q)_{exp}$ of phonons obtained from [17].

type SIB do not have fixed locations like those in solids; in spite of their nearly localized positions, they remain free to move in order of their locations, (ii) the excitation wave length λ_{ex} for $Q > 2\pi/d$ becomes shorter than d implying that the energy and momentum of the excitation can be carried by a single particle, and (iii) a particle of $\lambda_{ex} < \sigma$ (*i.e.*, $Q > 2\pi/\sigma$), can be expected to behave almost like a free particle since it does not have any wave superposition even with its nearest neighbor.

4. Concluding Remarks

The energy of elementary thermal excitations of liquid helium-4 obtained by using $E(Q) = \hbar^2 Q^2/4mS(Q)$ (concluded from our recent microscopic theory of a SIB [16]) matches closely with $E(Q)_{exp}$ of $He-II$ when $S(Q)$ data set used for this purpose conforms to $S(0) = 0$ which is concluded to be an important aspect of a superfluid at $T = 0$ [18]; interestingly, an important conclusion of our theory [16] that the relative configuration of particles in the superfluid state of a SIB is independent of T implies that $S(0) = 0$ should be valid for all $T \leq T_\lambda$ and this is corroborated by the fact that $E(Q)_{exp}$ of $He-II$ hardly change with T. Our theory also explains the anomalous nature of low Q phonons in $He-II$. Since the excitation spectrum of Bose condensed dilute gases too is found to be qualitatively identical to that of $He-II$ [23], our theory can similarly explain the related properties of BEC state of such gases. We note that $He-I$ and $He-II$ as per our theory differ from each other in relation to the relative configuration of their particles whose details are discussed in [16]. However, the important difference of $He-I$

and $He-II$ which affects $E(Q)$ can be identified with the collisional motion of particles in $He-I$ and collision-less state of particles in $He-II$ [16]. Consequently, $E(Q)$ of the two phases is expected to differ significantly not in the positions of their excitations but in their line-widths which should be much larger for $He-I$ than for $He-II$ and this agrees with experiments. In addition this study shows that: (i) $E(Q)_{exp}$ values of $He-II$ are consistent with $S(0)=0$ (not with $S(0)\neq 0$) and (ii) $E(Q)_{Fyn}$ (Eqn. 1) either equals $\approx 2E(Q)_{exp}$ at all Q if $S(Q)$ data conform to $S(0)=0$ or its apparent agreement with $E(Q)_{exp}$ at low Q seen for $S(0)\neq 0$ is plagued with *three problems* concluded in Section 2. As such all these facts provide good evidence of the accuracy of Eqn. 4 which implies that our theory provides strong foundation to understand the thermodynamic properties of a SIB. We would use our theory for a similar analysis of the hydrodynamic behavior of a SIB in our future course of studies. It should be interesting to note that our theory, for the first time, provided highly accurate account of several properties of $He-II$, *e.g.* : (i) logarithmic singularity of specific heat of $He-II$ and related properties, (ii) quantized vortices with an appropriate answer to the question raised by Wilks [22] on the validity of Feynman's explanation for their origin, (iii) T^3 dependence of specific heat, (iv) two fluid behavior with a conclusion that each particle simultaneously participates in superfluid and normal components, (v) superfluidity of quasi 1-D and 2-D systems.

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Appendix I

Data pertaining to a comparative study of $E(Q)_{mo}$ and $E(Q)_{exp}$. The Q is in $\mathring{A}^{-1}$ and $E(Q)$ in ^{o}K

Q	$S(Q)_{exp}$	$S(Q)'$	$\Delta S(Q)$	$E(Q)_{exp}$	$E(Q)_{mo}$	$E(Q)^{*}_{mo}$	$E(Q)_{eq5}$
0.00	0.0510	0.0000	0.0510	0.000	0.000	0.000	0.00
0.05	0.0583	0.0076	0.0500	0.804	0.129	1.037	0.92
0.10	0.0659	0.0173	0.0486	1.747	0.459	2.032	1.86
0.15	0.0736	0.0252	0.0484	2.757	0.926	3.015	2.81
0.20	0.0815	0.0321	0.0494	3.772	1.487	3.971	3.75
0.25	0.0897	0.0397	0.0500	4.772	2.111	4.891	4.68
0.30	0.0980	0.0474	0.0506	5.749	2.782	5.799	5.59
0.35	0.1065	0.0554	0.0511	6.694	3.469	6.684	6.47
0.40	0.1152	0.0632	0.0520	7.598	4.215	7.547	7.32
0.45	0.1242	0.0710	0.0532	8.452	4.940	8.378	8.13
0.50	0.1333	0.0818	0.0515	9.249	5.690	9.199	8.89
0.55	0.1427	0.0918	0.0509	9.979	6.409	9.990	9.62
0.60	0.1522	0.1025	0.0497	10.641	7.170	10.773	10.29
0.65	0.1620	0.1139	0.0481	11.237	7.900	11.527	10.92
0.70	0.1720	0.1229	0.0491	11.767	8.630	12.264	11.50
0.75	0.1822	0.1393	0.0429	12.232	9.360	12.984	12.02
0.80	0.1928	0.1535	0.0393	12.633	10.040	13.668	12.48
0.85	0.2040	0.1688	0.0352	12.971	10.730	14.301	12.89
0.90	0.2164	0.1852	0.0312	13.248	11.362	14.831	13.23
0.95	0.2303	0.2032	0.0271	13.464	11.880	15.243	13.51
1.00	0.2463	0.2225	0.0238	13.620	12.316	15.506	13.72
1.05	0.2648	0.2435	0.0213	13.717	12.606	15.616	13.86
1.10	0.2862	0.2664	0.0198	13.757	12.820	15.580	13.92
1.15	0.3109	0.2916	0.0193	13.740	12.880	15.410	13.90
1.20	0.3395	0.3192	0.0203	13.667	12.830	15.116	13.81
1.25	0.3716	0.3496	0.0220	13.540	12.720	14.759	13.65
1.30	0.4066	0.3833	0.0233	13.359	12.580	14.393	13.42
1.35	0.4438	0.4206	0.0232	13.125	12.430	14.051	13.12
1.40	0.4825	0.4625	0.0200	12.839	12.290	13.756	12.76
1.45	0.5219	0.5096	0.0123	12.503	12.204	13.521	12.33
1.50	0.5614	0.5625	-0.0011	12.118	12.150	13.350	11.85
1.55	0.6049	0.6232	-0.0183	11.684	12.030	13.135	11.30
1.60	0.6633	0.6926	-0.0293	11.202	11.690	12.662	10.70
1.65	0.7466	0.7724	-0.0258	10.680	11.040	11.852	10.04
1.70	0.8529	0.8623	-0.0094	10.148	10.260	10.914	9.34

Appendix I contd.

Q	$S(Q)_{exp}$	$S(Q)'$	$\Delta S(Q)$	$E(Q)_{exp}$	$E(Q)_{mo}$	$E(Q)^*_{mo}$	$E(Q)_{eq5}$
1.75	0.9721	0.9626	0.0095	9.644	10.069	9.706	
1.80	1.0940	1.0671	0.0269	9.204	9.408	9.132	
1.85	1.2100	1.1784	0.0316	8.866	8.942	8.693	
1.90	1.3080	1.2616	0.0464	8.667	8.697	8.433	
1.95	1.3800	1.3334	0.0466	8.644	8.664	8.421	
2.00	1.4180	1.3684	0.0496	8.833	8.861	8.601	
2.05	1.4240	1.3736	0.0504	9.271	9.269	9.030	
2.10	1.4050	1.3443	0.0607	9.941	9.863	9.337	
2.15	1.3690	1.2992	0.0698	10.784	10.621	10.290	
2.20	1.3230	1.2497	0.0733	11.742	11.523	11.168	
2.25	1.2730	1.2031	0.0699	12.751	12.546	12.135	
2.30	1.2260	1.1657	0.0603	13.753	13.634	13.084	
2.35	1.1800	1.1391	0.0409	14.687	14.813	14.228	